Fully Developed Viscous—Flow Heat Transfer in Curved Circular Tubes with Uniform Wall Temperature

Fully developed forced-convective heat transfer to viscous flow of a constant-property Newtonian fluid in curved circular tubes was studied theoretically for a uniform-wall-temperature boundary condition. Numerical solutions were obtained over a wide range of Prandtl numbers for Dean numbers as large as 1,200. The curvature ratio (radius of bend/inside radius of tube) was included as an independent parameter and, for values as small as 10, was found to have a negligible effect on the peripherally averaged Nusselt number. The average Nusselt numbers for the important Prandtl number range of 0.7 to 5 were correlated by $N_{\overline{Nu}} = 0.836\ N_{De}^{0.5}\ N_{Pr}^{0.1}$ for $N_{De} \geqq 80$.

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SCOPE

Curved tubes and passages are employed in many heat transfer applications. For example, helical and spiral coils are often used for transferring heat in mixing, storage, and reactor vessels, as well as in process heat exchangers. Curved heat transfer passages have been used in rocket engines and in jacketed vessels. Geometrical restrictions often dictate that curved tube configurations be used instead of straight tubes. However, it is now apparent that curved tubes provide, in many instances, superior heat transfer performance compared with straight tubes.

There are also applications where curved flow passages can be used to improve mass transfer rates, such as in membrane blood oxygenators (Weissman and Mockros, 1968), in kidney dialysis devices (Dravid, 1969), and in reverse osmosis units (Srinivasan and Tien, 1971). In addition to inducing higher average heat and mass transfer coefficients, the secondary flows present in curved tube flow channels cause a marked variation in local transfer coefficients around the periphery of the channel. A knowledge of this variation may suggest advantages in instances where only a portion of the peripheral boundary is available for transport processes.

Fully developed curved tube heat transfer under the boundary condition of axially uniform wall heat flux with peripherally uniform wall temperature has been studied both experimentally and theoretically (Dravid et al., 1971; Akiyama and Cheng, 1971; Kalb and Seader, 1972). The boundary condition of uniform wall temperature has received less attention although it is more important in applications (including the analogous mass transfer boundary condition). Almost all theoretical investigations have assumed that the ratio of the radius of curvature of the tube to the inside radius of the tube—the curvature ratio R/a—is large.

Maekawa (1964) obtained a perturbation solution limited to low Dean numbers N_{De} where $N_{De} = N_{Re}/(R/a)^{1/2}$. Mori and Nakayama (1965, 1967) obtained solutions at high Dean numbers for the above boundary conditions using a boundary-layer/potential-flow method; their results have been shown to be inaccurate except for Prandtl numbers near one (Dravid et al., 1971; Akiyama and Cheng, 1971, 1972). Akiyama and Cheng's (1972) numerical solution for uniform wall temperature included Dean numbers as large as 200 but was restricted to large curvature ratios.

The numerical solutions of Dravid et al. (1971) and Tarbell (1972) for heat transfer in the thermal entrance region of curved tubes showed that the Nusselt number behaves in an oscillatory manner. These oscillations, caused by the interaction of the secondary flow with the developing thermal boundary layer, damp out as the fully developed temperature field is approached. The fully developed Nusselt number is approached within relatively short axial distances from the tube entrance, and Dravid et al. (1971) recommend that entrance effects be neglected and that the fully developed Nusselt number be used for design.

An objective of this study was to obtain numerical solutions for fully developed heat transfer with a uniform-wall-temperature boundary condition for a much wider range of Dean numbers and Prandtl numbers than studied previously. Also, the influence of the curvature ratio on heat transfer was investigated.

CONCLUSIONS AND SIGNIFICANCE

Numerical solutions for fully developed forced-convective heat transfer to viscous flow of a constant-property Newtonian fluid in curved circular tubes with a uniform-wall-temperature boundary condition were obtained for Dean numbers as large as 1,200. Prandtl numbers and curvature ratios were varied from 0.05 to 1,600 and from 10 to 100, respectively. Results for the peripherally averaged Nusselt number are shown in Figures 5

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and 6. For low Dean numbers, the results are in excellent agreement with the perturbation solution of Maekawa (1964). For high Dean numbers, at a Prandtl number of 0.7, the results are in fair agreement with the approximate solution of Mori and Nakayama (1967). For Nusselt numbers less than about six, the numerical computations agree well with the correlation of Akiyama and Cheng (1972) based on numerical calculations for large curvature ratios. However, for larger Nusselt numbers, the extrapolation of Akiyama and Cheng's numerical computations through the use of their similarity parameter $N_{De}^2\ N_{Pr}$ diverges from the numerical results of this study. A dimensional analysis led to the conclusion that this similarity parameter has a limited range of usefulness.

The results indicate that, for curvature ratios as small as 10, essentially all of the effect of the curvature ratio on the peripherally averaged Nusselt number is incorporated into the effect of the Dean number, as shown in Figures 5 and 6. However, an additional, but not large, effect of the curvature ratio, above that expressed in the Dean number, was found for the peripheral variation of the Nusselt number, as shown in Figure 8.

In the moderate Prandtl-number range of 0.7 to 5—including gases, many common liquified gases, and many low-viscosity liquids—and at high Dean numbers, the results of this study are correlated well by Equation (11). Compared to a similar equation (Kalb and Seader, 1972) for the boundary condition of axially uniform wall heat flux with peripherally uniform wall temperature, the present results show a smaller dependence on the Prandtl number.

Large increases in mass transfer coefficients over those attained in straight tubes are predicted for applications involving high Schmidt numbers. The conclusion is drawn that any bioengineering mass transfer situation involving viscous flow in curved tubes, whether in vivo or in vitro, cannot be described accurately by a straight-tube model if the Schmidt number is large, even when the tubes are only slightly curved.

FORMULATION OF THE PROBLEM

The fully developed velocity profiles used here are based on the full continuity and Navier-Stokes equations for viscous flow of a constant-property fluid in a toroidal geometry. These equations were solved numerically using methods developed by Austin and Seader (1973). The assumption of large curvature ratio ($R/a >> 1$) was not imposed. The parameters for the velocity profiles used in this study and the calculated ratio of friction factors for curved and straight tubes are given in Table 1.

With reference to the orthogonal, toroidal coordinate system shown in Figure 1, the dimensional thermal energy equation for steady state viscous flow of a constant property fluid, when viscous dissipation and axial conduction of heat are neglected, is

$$u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \theta} + \frac{w}{R + r\sin\theta}\frac{\partial T}{\partial \psi} = \frac{\nu}{N_{Pr}}\left[\frac{\partial^2 T}{\partial r^2} + \left(\frac{1}{r} + \frac{\sin\theta}{R + r\sin\theta}\right)\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\cos\theta}{r(R + r\sin\theta)}\frac{\partial T}{\partial \theta}\right] \quad (1)$$

For transport processes in curved circular tubes, allowance must be made for proper averaging of certain quantities around the tube periphery. For example, the peripherally averaged Nusselt number at any axial position is given by

$$N_{\overline{Nu}} = \frac{2\bar{h}a}{k} = \frac{1}{2\pi(R/a)}\int_0^{2\pi} N_{Nu}\,(R/a + \sin\theta)\,d\theta \quad (2)$$

For a uniform-wall-temperature boundary condition, the following relation applies when the temperature field is fully developed:

$$\frac{\partial T}{\partial \psi} = \left(\frac{T_w - T}{T_w - T_m}\right)\frac{dT_m}{d\psi} \quad (3)$$

It is convenient to define the dimensionless temperatures

$$G = \frac{(T_w - T)(R/a)}{N_{Re}\,N_{Pr}\,dT_m/d\psi}\left(\frac{T_w - T_m}{T_w - T_c}\right) \quad (4)$$

and

$$\phi = \frac{T_w - T}{T_w - T_m} = \frac{G}{G_m} \quad (5)$$

where T_c, the temperature at the center line of the tube, has been utilized in the definition of G. Introducing the dimensionless temperature G, as well as dimensionless

TABLE 1. PARAMETERS FOR VELOCITY PROFILES AND CALCULATED FRICTION FACTORS*

Run	R/a	N_{De}	f_c/f_s
1	100	1.000	1.0001
2	100	1.937	1.0001
3	100	4.998	1.0005
4	20	9.723	1.006
5	100	12.39	1.009
6	10	15.34	1.031
7	100	21.36	1.053
8	20	24.92	1.088
9	10	40.13	1.220
10	20	49.82	1.268
11	10	60.24	1.344
12	10	80.24	1.443
13	20	91.11	1.473
14	100	100.3	1.495
15	10	100.8	1.538
16	15.2	103.2	1.533
17	20	150.0	1.724
18	20	253.2	2.098
19	10	332.7	2.376
20	15.2	390.4	2.507
21	100	401.8	2.489
22	15.2	511.5	2.804
23	15.2	714.3	3.248
24	15.2	898.4	3.630
25	15.2	1,204	4.256

* Some of the velocity profiles for R/a of 100 were calculated by Austin and Seader (1973).

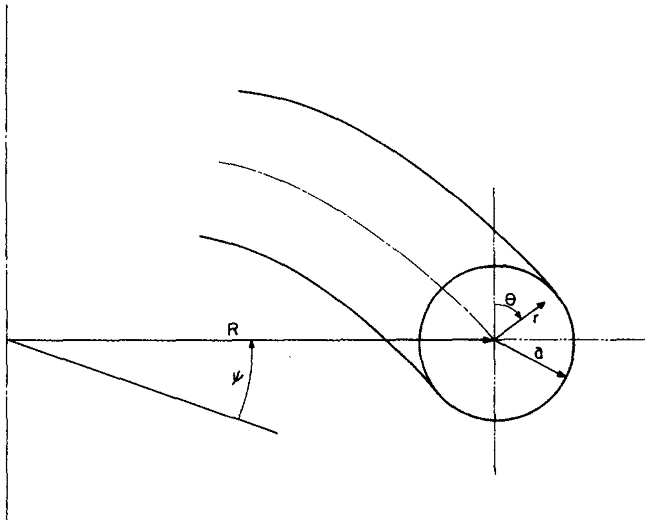

Fig. 1. Toroidal coordinate system for a curved circular tube.

velocities and a dimensionless radial coordinate into Equation (1), and making use of Equation (3), yields

$$\frac{\partial^2 G}{\partial \eta^2} + \frac{1}{\eta^2}\frac{\partial^2 G}{\partial \theta^2} + \left(\frac{1}{\eta} + \frac{\sin\theta}{R/a + \eta \sin\theta} - N_{Pr}\, U\right)\frac{\partial G}{\partial \eta}$$
$$+ \left(\frac{\cos\theta}{\eta(R/a + \eta\sin\theta)} - N_{Pr}\frac{V}{\eta}\right)\frac{\partial G}{\partial \theta}$$
$$+ \frac{W(R/a)}{(R/a + \eta\sin\theta)N_{Re}} \cdot \frac{G}{G_c} = 0 \quad (6)$$

with a boundary condition of $G = 0$ at the wall of the tube. The numerical solution of Equation (6) and the reason for introducing T_c (and therefore G_c) are discussed in the next section.

The local Nusselt number is obtained by considering the local heat flux at the wall. In terms of dimensionless temperatures

$$N_{Nu} = \frac{2ha}{k} = \frac{-2(\partial G/\partial \eta)_{\eta=1}}{G_m} = -2\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=1} \quad (7)$$

The bulk-temperature gradient $dT_m/d\psi$ is related to the peripherally averaged heat flux at the wall $\overline{q}$ through a heat balance on the flowing fluid, neglecting axial conduction. The peripherally averaged Nusselt number is then obtained in terms of G as

$$N_{\overline{Nu}} = \frac{2\overline{h}a}{k} = \frac{1}{2\,G_c} \quad (8)$$

The flow-averaged value of G is defined by

$$G_m = \frac{1}{W_m \pi}\int_0^{2\pi}\int_0^1 W\,G\,\eta\, d\eta\, d\theta$$
$$= \frac{2}{\pi N_{Re}}\int_0^{2\pi}\int_0^1 W\,G\,\eta\, d\eta\, d\theta \quad (9)$$

NUMERICAL-SOLUTION PROCEDURE

The dimensionless thermal energy equation, Equation (6), contains the factor G/G_c in one of the terms. Kalb and Seader's (1972) analysis for axially uniform wall heat flux with peripherally uniform wall temperature led to a dimensionless thermal energy equation without a temperature ratio but otherwise identical to Equation (6). This similarity suggested a numerical procedure for solving Equation (6).

Equation (6) was transformed to a finite difference form and solved by a double iterative procedure. To start the calculations, assumed values of the temperature field G/G_c were assigned to each grid point. It was convenient to use the constant-heat-flux solutions of Kalb and Seader (1972) for this initial approximation. A point successive overrelaxation (S.O.R.) method was then employed, with iterative updating of the factor G/G_c. The finite difference grid and spacing, S.O.R. procedure, relaxation factors, and convergence criteria were the same as described by Kalb and Seader (1972). Full particulars on numerical procedures and results for both of the boundary conditions mentioned above are available (Kalb, 1973).

The term T_c was introduced more or less arbitrarily into Equation (4). Other temperatures could have been used. However, had the flow-averaged mean temperature T_m been used instead of T_c, the factor G/G_m would have had to be reevaluated after every iteration in the numerical procedure outlined above. This would have led to much longer computation times since G_m was evaluated by numerical integration of the double integral in Equation (9). Using T_c as a reference temperature avoided this problem. Also, the converging radial grid lines used in the numerical solution provided a high grid-point density in the vicinity of the center line.

To provide a check on the consistency and accuracy of the numerical method, calculations were performed using a parabolic velocity profile and a value of 10^{16} for R/a, which approximates the infinite curvature of a straight tube. The calculated Nusselt number was 3.651, in good agreement with the theoretical value, 3.657, and the temperature field was axially symmetric, as expected.

Another check on the accuracy of the numerical solution was made. Equation (7) was combined with Equation (2) to give

$$N_{\overline{Nu}} = \frac{-1}{\pi(R/a)}\int_0^{2\pi}\left(\frac{\partial \phi}{\partial \eta}\right)_{\eta=1}(R/a + \sin\theta)\, d\theta \quad (10)$$

Equations (8) and (10) constitute two independent means of calculating the average Nusselt number from the numerical data. The integrals were evaluated numerically by one- and two-dimensional Simpson's rules. The percentage difference in the two estimates of $N_{\overline{Nu}}$ was taken as a measure of error for solution of the thermal energy equation, assuming that the velocity profiles were exact. For 103 runs, the error in the heat transfer results predicted by Equations (8) and (10) was 0.7, 1.1, and 3.6%, respectively, for the average, standard, and maximum deviations. All average Nusselt numbers reported here are based on Equation (8). An estimate of the accuracy of the velocity profiles was made by calculating the peripherally averaged friction factor by two independent procedures (Kalb and Seader, 1972), with the conclusion that, except for the very high Dean-number region, the average Nusselt numbers appear to be accurate to within 2%.

TEMPERATURE PROFILES

Figures 2 and 3 are isometric plots of the dimensionless temperature. Figure 2, for a Dean number of 512 and a Prandtl number of 0.7, indicates that the temperature field is skewed strongly toward the outer wall or away from center of curvature of the tube. Akiyama and Cheng (1972) reported a numerically calculated temperature field for a Dean number of 123 which is similar to that

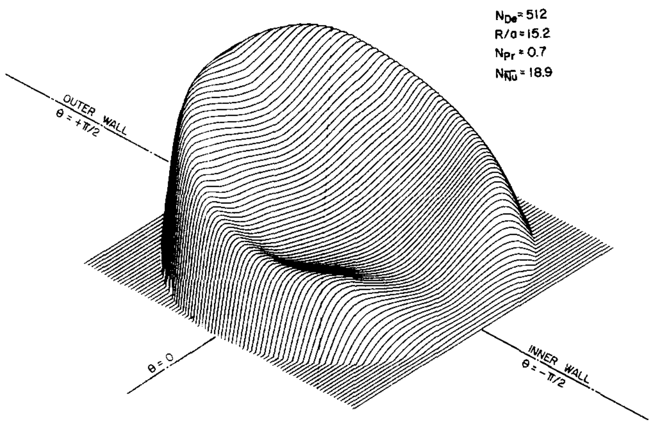

Fig. 2. Dimensionless temperature for N_{De} = 512, R/a = 15.2, and N_{Pr} = 0.7.

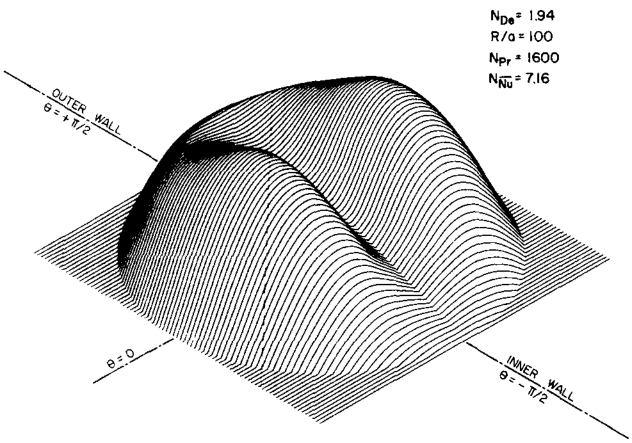

Fig. 3. Dimensionless temperature for N_{De} = 1.94, R/a = 100, and N_{Pr} = 1,600.

of Figure 2. Mori and Nakayama's (1967) analysis does not predict temperature distributions like those in Figures 2 and 3; instead, that analysis predicts that temperatures in the central flow region (their potential-flow core) are constant along lines perpendicular to the radius of curvature of the tube.

The thermal boundary condition was found to have a significant effect on the temperature field, in agreement with the calculations of Akiyama and Cheng (1972). Figure 4 compares the results for N_{De} = 512 and N_{Pr} = 0.7 with the corresponding temperature profiles obtained for axially uniform wall heat flux with peripherally uniform wall temperature (Kalb and Seader, 1972). The most striking difference in the temperature profiles, with a Prandtl number of 0.7, is the increased degree of concavity in the central region of the tube for the uniform-wall-temperature case. Also, there is a greater spread in the local temperature derivatives at the wall (and, therefore, local Nusselt numbers) from inner to outer walls for the uniform-wall-temperature boundary condition. These differences are less marked at higher Prandtl numbers than 0.7.

Schmidt numbers for mass transfer in liquids are commonly about 1,000. Figure 3 illustrates the large effects predicted for such systems and can be interpreted as a dimensionless concentration profile for N_{De} = 1.94 and N_{Sc} = 1,600. Even at this very low Dean number, the average mass transfer Nusselt number is about 100% larger than the straight-tube value, 3.657. The secondary circulation, even though slight, has a large effect on mass transfer coefficients and concentration profiles when the Schmidt number is large.

AVERAGE AND LOCAL NUSSELT NUMBERS

Maekawa (1964) utilized the perturbation velocity profiles of Dean (1928) and obtained perturbation solutions of the thermal energy equation at large curvature ratios for boundary conditions of both uniform wall temperature and of axially and peripherally uniform wall heat flux. The numerically calculated average Nusselt numbers obtained in this study for the low-Dean-number region are compared with Maekawa's solution for uniform wall temperature in Figure 5. The perturbation solution diverges to large negative values of $N_{\overline{Nu}}$ just to the right of the points where the perturbation-solution curves are terminated in Figure 5.

The numerical results are compared with several other uniform-wall-temperature theoretical studies in Figure 6, where Maekawa's solution was used to fill in the curves for Dean numbers less than one. Akiyama and Cheng (1972) solved the thermal energy equation in the thermal entrance region for large curvature ratios under the uniform-wall-temperature boundary condition. They reported only fully developed Nusselt numbers and did not obtain solutions for Dean numbers greater than 200. However, they proposed an equation for use over the entire range of viscous flow conditions. Their equation is plotted in Figure 6 for Prandtl numbers of 5, 100, and 1,600; convergence of these curves with the numerical results of this study for $N_{\overline{Nu}}$ less than about six is good. However, Akiyama and Cheng's equation eventually diverges from the results of this study. A possible explanation for this is

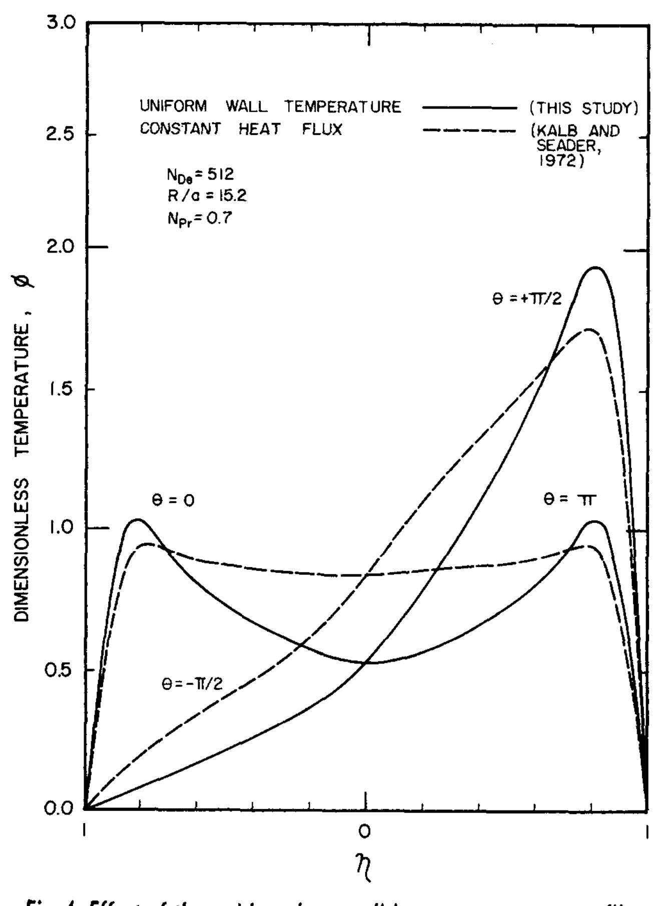

Fig. 4. Effect of thermal boundary conditions on temperature profiles.

that Akiyama and Cheng may have extrapolated prematurely to high Dean numbers their numerical results for Dean numbers less than 200. Also, it is shown in the next section that Akiyama and Cheng's similarity parameter $N_{De}^2\, N_{Pr}$ has only a relatively narrow range of usefulness. Akiyama and Cheng's (1971) extrapolation for the constant-heat-flux boundary condition is similarly not in agreement with either the numerical results of Kalb and Seader (1972) or with the empirical correlation of Dravid et al. (1971). Dravid et al.'s (1971) numerical datum of $N_{\overline{Nu}} = 16$ at $N_{Pr} = 5$ and $N_{De} = 225$ is also shown in Figure 6.

Mori and Nakayama's (1967) boundary-layer/potential-flow analysis for uniform wall temperature was not carried to the same degree of approximation as their constant-heat-flux analysis; consequently they were unable to predict any effect of the boundary conditions on the fully developed Nusselt number. Their equation is plotted for $N_{Pr} = 0.7$ in Figure 6; there is only fair agreement with the results of this study.

Tarbell (1972) reported the asymptotic values of the average Nusselt number attained in his thermal-entrance-region solutions. Although his data are for different Prandtl numbers, the results are consistent with those of this study as indicated in Figure 7. Tarbell's data for $N_{Pr} = 0.1$ are close to the results of this study for $N_{Pr} = 0.05$; this is consistent with the observation that the Prandtl-number dependence is small at low Prandtl numbers (Kalb and Seader, 1972).

The method of Hellums and Churchill (1964) for finding the minimum parametric description of a mathematical model was applied to the partial-differential equations which describe fully developed viscous flow and heat transfer in a toroidal geometry. Their procedure predicts a dependence of $N_{\overline{Nu}}$ on N_{De}, R/a, and N_{Pr}. With a developing temperature field, the Graetz number $\dot{m}c_p/kz$ enters as an additional parameter. The assumption that R/a is $>> 1$ leads to simplified equations, with R/a eliminated as an independent parameter. It was an objective of the present study to determine the influence of R/a above that expressed in the Dean number $N_{Re}/(R/a)^{1/2}$.

Figures 5 and 6 indicate that the curvature ratio, in the range of 10 to 100, has no discernible effect on the average Nusselt number. However, the curvature ratio has a definite effect on the peripheral variation of the fully developed Nusselt number. Figure 8 shows this effect for two velocity profiles having very nearly the same Dean number but curvature ratios of 10 and 100. The data

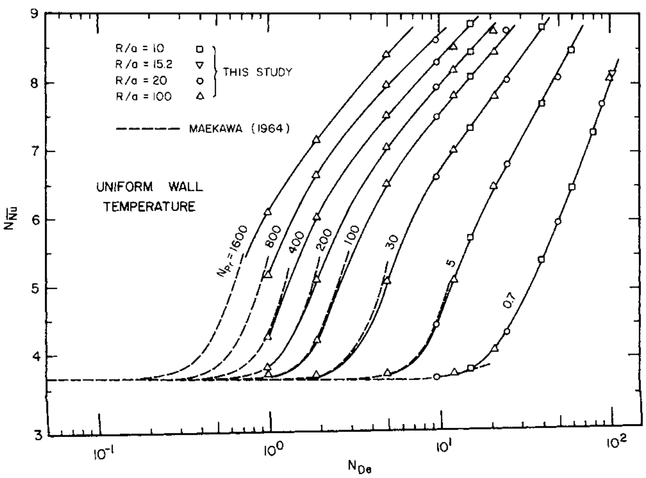

Fig. 5. Comparison of numerically calculated average Nusselt numbers with Maekawa's perturbation solution.

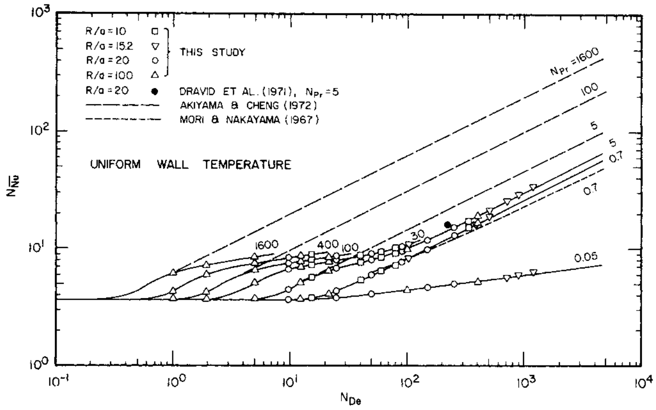

Fig. 6. Comparison of numerically calculated average Nusselt numbers with other theoretical studies.

shown for Prandtl numbers of 0.7 and 5 indicate that increasing the curvature ratio while maintaining a constant Dean number increases the spread in the local Nusselt numbers from inner to outer walls; however, this effect is apparently not a large one.

Figure 8 indicates that local Nusselt numbers in the vicinity of the inner wall are less than the straight-tube value, 3.657. This is a result of two factors: (1) for curved-tube flow, axial velocities near the inner wall are usually smaller than those in a straight tube; and (2) temperatures of the secondary flow entering the central-flow region near the inner wall are close to the wall temperature, causing radial temperature gradients to be smaller. These effects increase as the Dean number increases, causing the Nusselt number at the inner wall to decrease until a limiting value is apparently reached. Local Nusselt numbers at the outer wall, however, continue to increase as the Dean number increases. Exceptions to the above behavior occur sometimes at very low Dean numbers, especially when the curvature ratio is also small. Under these conditions, both Lorrain and Bonilla (1970) and Austin and Seader (1973) showed that the axial velocity can be skewed toward the inner wall instead of toward the outer wall; this caused the maximum local Nusselt number to occur at the inner wall for several runs of this study. This effect is more pronounced at lower Prandtl numbers, where secondary convective energy fluxes are less effective in influencing the shape of the temperature field. The effect of the Dean and Prandtl numbers on the local Nusselt numbers is qualitatively the same as found for constant heat flux (Kalb and Seader, 1972).

A concise correlation of the numerically calculated fully developed Nusselt numbers for the Prandtl number range of 0.7 to 5 is

$$N_{\overline{Nu}} = 0.836\, N_{De}^{0.5}\, N_{Pr}^{0.1} \qquad N_{De} \geqq 80 \tag{11}$$

with a standard deviation of 2.8% and a maximum deviation of 4.7%. Diffusional mass transfer of a chemical species in a gas stream often involves Schmidt numbers in the range of 0.7 to 5; Equation (11) applies in this instance with the substitution of N_{Sc} for N_{Pr}.

The Dean number exponent of 0.5 in Equation (11) is in agreement with the prediction of Mori and Nakayama (1965) at high Dean numbers. The Prandtl number exponent of 0.1, however, is less than values reported under the boundary condition of axially uniform wall heat flux with peripherally uniform wall temperature. For that boundary condition, the experimental data of Dravid et al.

(1971) indicated a Prandtl number exponent of 0.175, and the numerical data of Kalb and Seader (1972) indicated an exponent of 0.200. Tarbell's (1972) numerical results for uniform wall temperature support the lower Prandtl-number dependence observed in this study (see Figure 7).

SIMILARITY PARAMETER $N_{De}^2 N_{Pr}$

In their paper treating the constant-heat-flux boundary condition, Akiyama and Cheng (1971) described conditions under which the fully developed Nusselt number could be shown to be a function of a single new similarity parameter $N_{De}^2 N_{Pr}$. In addition to the restriction of large curvature ratios, it can be demonstrated that an assumption necessary to their development was the neglect of the inertial terms in the Navier-Stokes equations, with the exception of the centrifugal force terms due to flow in the axial direction. When all of the inertial terms are retained and the same dimensionless variables are used, it can be shown that N_{De} appears as an additional parameter in the equations; specifically, N_{De}^2 then enters as a multiplier of the previously neglected inertial terms. Therefore, the Nusselt number in the most general case must be considered a function of both N_{De} and N_{Pr}, independently. A similarity analysis using the Hellums-Churchill (1964) procedure leads to the same conclusion.

For both the constant-heat-flux and uniform-wall-temperature boundary conditions, Akiyama and Cheng (1971, 1972) proposed extrapolation equations based on their similarity parameter $N_{De}^2 N_{Pr}$. It should be made clear that Akiyama and Cheng's numerical solutions employed all of the inertial terms in the Navier-Stokes equations with the exception of those terms neglected due to the assumption of large curvature ratios. The analysis which follows is concerned with their constant-heat-flux correlation, since, for that case, there are several sets of experimental data available for comparison. The conclusions below apply also, however, to Akiyama and Cheng's (1972) results for uniform wall temperature.

Akiyama and Cheng found that their numerical Nusselt number data for $N_{Pr} \geqq 1$ fell on a single curve when plotted against $N_{De}^2 N_{Pr}$, or, equivalently, against $N_{De} N_{Pr}^{1/2}$. Except at $N_{Pr} = 1$, Akiyama and Cheng's numerical data have values of $(N_{De}^2 N_{Pr})^{1/2}$ less than about 100. Their extrapolation equation for constant-heat-flux conditions is plotted in Figure 9. For values of $(N_{De}^2 N_{Pr})^{1/2}$ less than about 100, the numerical results of Kalb and Seader (1972) for Prandtl numbers of 5, 30, and 400 fall close to the correlation of Akiyama and Cheng. The experimental data of Mori and Nakayama (1965) for $N_{Pr} = 0.71$ are in close agreement with Akiyama and Cheng.

Dravid et al.'s (1971) empirical correlation for the fully developed Nusselt number with axially uniform wall heat flux and peripherally uniform wall temperature is plotted in Figure 9 for N_{Pr} equal to 30 and 400; the resulting curves, when extrapolated, are consistent with the numerical results of Kalb and Seader (1972). The experimental data of Dravid et al. (1971) approximate closely the boundary condition of axially uniform wall heat flux with peripherally uniform wall temperature and are included in Figure 9.

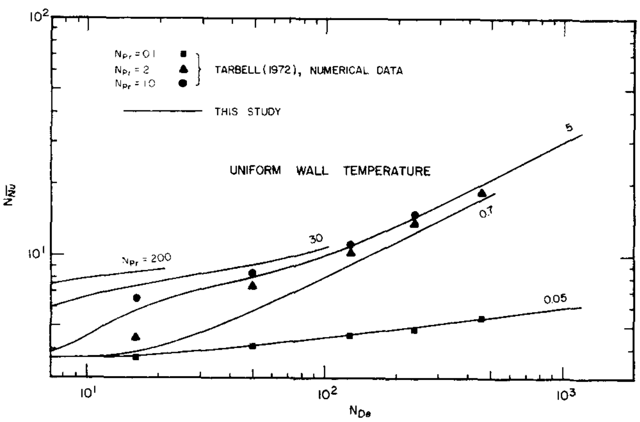

Fig. 7. Comparison of numerically calculated average Nusselt numbers with Tarbell's data.

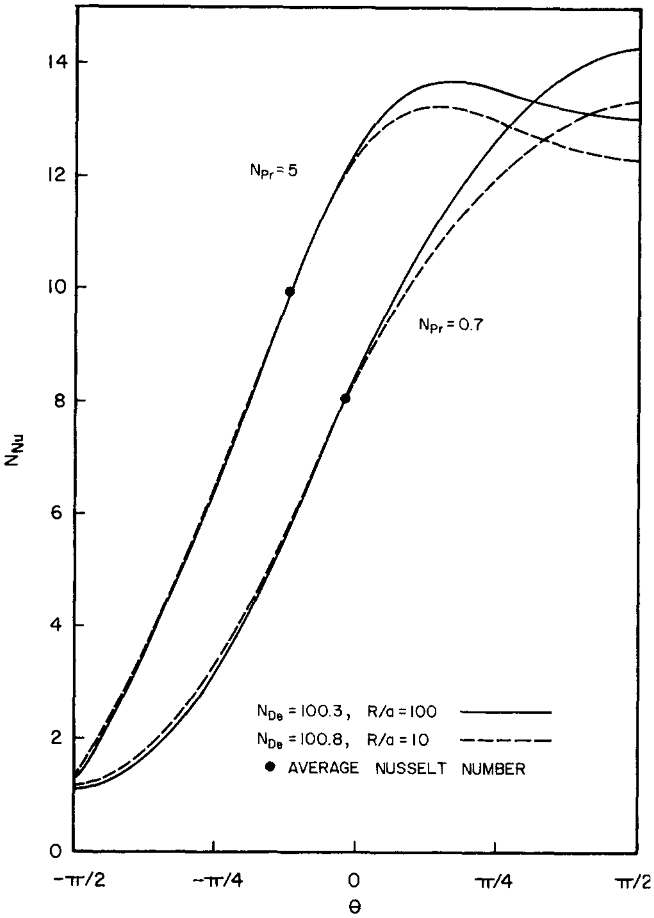

Fig. 8. Effect of curvature parameter above that expressed in the Dean number.

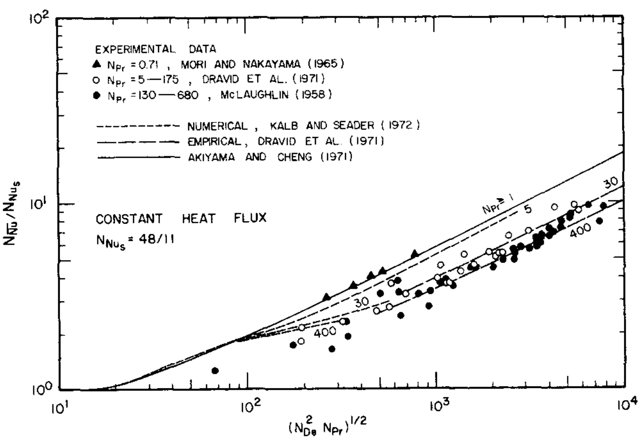

Fig. 9. Comparison of the Akiyama and Cheng correlation with other studies.

Seban and McLaughlin (1963) measured local heat transfer coefficients in an electrically heated coil. They reported that the wall temperature varied peripherally and that heat was conducted peripherally in the wall, making the exact nature of their boundary condition uncertain. They presented their results in terms of apparent heat transfer coefficients, calculated from the measured wall temperatures at the inside and outside of the tube bend and a wall heat flux based on the total power input to the coil. Heat transfer coefficients calculated in this manner must be corrected for heat conducted peripherally in the wall before they could be expected to approximate the true coefficients; nevertheless, these apparent coefficients have often been compared incorrectly with theory (Mori and Nakayama, 1965; Özisik and Topakoglu, 1968; Akiyama and Cheng, 1971). Although the thermal boundary condition is uncertain, all of the data presented by McLaughlin (1958) which were corrected for peripheral conduction of heat in the wall are included for comparison in Figure 9, rather than the uncorrected data of Seban and McLaughlin (1963).

The numerical results of Kalb and Seader (1972) and the experimental results of Dravid et al. (1971) and of McLaughlin (1958) indicate that the correlation of Akiyama and Cheng is probably a valid extrapolation only for Prandtl numbers near one. Also, at high Dean numbers, Figure 9 shows a definite effect of the Prandtl number over that expressed within $(N_{De}^2 N_{Pr})^{1/2}$, indicating a separate dependence on N_{De} and N_{Pr}.

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NOTATION

a = inside radius of tube
c_p = specific heat at constant pressure
D = diffusivity of a chemical species
f = friction factor
G = dimensionless temperature defined in Equation (4)
h = local heat transfer coefficient = $q/(T_w - T_m)$
$\overline{h}$ = peripherally averaged heat transfer coefficient = $\overline{q}/(T_w - T_m)$
k = thermal conductivity
$\dot{m}$ = flow rate (mass per unit time)
N_{De} = Dean number = $N_{Re}/(R/a)^{1/2}$
N_{Nu} = local Nusselt number = $2ha/k$
$N_{\overline{Nu}}$ = peripherally averaged Nusselt number = $2\overline{h}a/k$
N_{Nu_s} = straight-tube Nusselt number, equal to 3.657 for uniform wall temperature and 48/11 for constant heat flux
N_{Pr} = Prandtl number = $c_p\mu/k$
N_{Re} = Reynolds number = $2aw_m/\nu$
N_{Sc} = Schmidt number = ν/D
q = local heat flux at the wall
$\overline{q}$ = peripherally averaged heat flux at the wall
r = radial coordinate in the tube cross section
R = radius of curvature of the tube at the center line
T = temperature
u, v, w = velocity components in r, θ, ψ directions, respectively
U, V, W = dimensionless velocity components in η, θ, ψ directions, given respectively by ua/ν, va/ν, wa/ν
z = axial distance along center line of tube, $R\psi$

Greek Letters

η = dimensionless radial coordinate = r/a
θ = angular coordinate in the tube cross section
μ = viscosity
ν = kinematic viscosity = μ/ρ
ρ = density
ϕ = dimensionless temperature defined in Equation (5)
ψ = angular coordinate normal to the tube cross section

Subscripts

c = value at center line; also curved tube
m = flow-averaged mean value
s = straight tube
w = value at the wall

Superscript

— = peripherally averaged value

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